

# Fermion-induced quantum critical points in 3D Weyl semimetals

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Fermion-induced quantum critical points (FIQCP) were recently discovered at the putatively-first-order transitions between 2D Dirac semimetals and the Kekule valence bond solids (Kekule-VBS) on the honeycomb lattice by *sign-free* quantum Monte Carlo (QMC) simulations in arXiv:1512.07908. Such FIQCPs realize type-II Landau-forbidden transitions, in contrast to deconfined quantum critical points (DQCP) which are type-I. Here, we present renormalization group (RG) analysis of possible FIQCP in 3D topological Weyl semimetals at a  $Z_3$  symmetry-breaking transition that is putatively-first-order according to the Landau criterion. Interestingly, we find that FIQCPs occurs at the  $Z_3$  nodal-nematic transitions in 3D double-Weyl semimetals (monopole charges  $\pm 2$ ), where the cubic terms are irrelevant and an enlarged  $U(1)$  symmetry emerges at low-energy. To the best of our knowledge, this is the *first* example of Landau-forbidden quantum transitions in 3+1D.

**Introduction:** The nature of a quantum phase transition is strongly dictated by the symmetry of order parameters and spatial dimensions of the systems in question [1]. One textbook criterion according to Landau [2, 3] states that if cubic terms of order parameters form a trivial representation of the symmetry group of the systems, the phase transition is necessarily first-order. It is easiest to be seen from the fact that the order parameter will develop a finite jump through phase transition, if the Landau-Ginzburg (LG) free energy includes cubic terms of order parameters. Previous work showed that this mean-field criterion works well in three dimensions (3D) or higher [4–6]. When this Landau’s cubic-term criterion is violated due to the fluctuations beyond mean-field theories, such novel phase transitions are dubbed as type-II Landau-forbidden transitions [7]. Note that a type-I Landau-forbidden transition refers to a continuous phase transition that occurs between two symmetry-incompatible phases; a novel scenario of type-I Landau-forbidden phase transitions is provided by deconfined quantum critical points (DQCP) [8–17].

One naturally wonders where can type-II Landau-forbidden transitions occur. Since fluctuations beyond mean-field theories may open the possibilities beyond the Landau cubic-term criterion, one scenario of type-II Landau-forbidden transitions was provided by strong fluctuations in low dimensions: the quantum three-state Potts model in 1+1D (equivalently classical three-state Potts model in 2D) is an exactly solvable model providing a well-known example which violates the Landau’s cubic-term criterion [18]. In higher dimensions, a scenario of realizing a type-II Landau-forbidden transition was introduced recently [7]: quantum phase transitions in fermionic systems. At zero temperature, gapless fermionic degrees of freedom must be retained in the quantum LG theory and the presence of them at quantum phase transitions may dramatically change the nature of critical behaviors. Although modifications of critical behaviors by gapless fermions have been extensively studied [19–32] it was only shown recently in Ref.

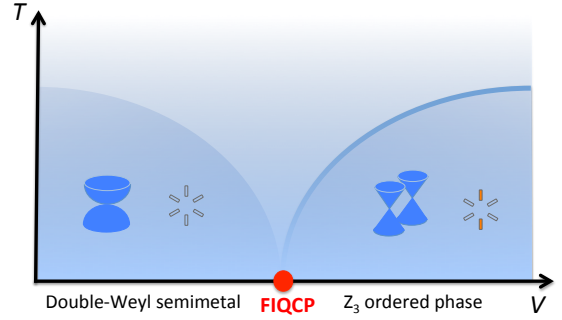


FIG. 1. A schematic phase diagram for a  $Z_3$  nodal-nematic transition from a topological double-Weyl semimetal to a nematic phase where each double-Weyl point splits into two Weyl points. A FIQCP emerges at zero temperature while the transition at finite temperature is still first-order.

[7] by both large-scale Majorana quantum Monte Carlo (QMC) [33, 34] simulations and large- $N$  renormalization group (RG) analysis that gapless Dirac fermions can drive a putative first-order quantum phase transition between 2D Dirac semimetals and the Kekule valence bond solids (Kekule-VBS) into a continuous one, which is called fermion-induced quantum critical point (FIQCP). Such FIQCP was also confirmed by a more recent RG analysis using  $\epsilon$ -expansion [35].

When symmetry-breaking happens in a system with gapless fermions, there are various fates of fermions. For instance, the Kekule-VBS order breaks translation symmetry, and gaps out Dirac fermions in the ordered phase [7, 36–41]. A nodal-nematic order, on the other hand, does not gap out nodal fermions but shifts positions of nodes in  $k$ -space [31, 32]. Here, we investigate if FIQCP can occur at a  $Z_3$  nodal-nematic phase transition in 3D topological double-Weyl semimetal [42–47], where  $Z_3$  order parameter cannot gap out the fermions owing to non-vanishing monopole charge ( $\pm 2$ ) of double-Weyl points. Instead, when nematic orders form, each double-Weyl point splits into two Weyl points with monopole charge  $\pm 1$  [48–54], breaking rotational symmetry from  $C_6$  to

$C_2$ . At such transition, cubic terms of order-parameter are allowed in quantum LG free energy; nonetheless, we show that the putative first-order phase transition can be driven into a continuous one, i.e., a FIQCP. A schematic phase diagram for the occurring of such a FIQCP is shown in Fig. 1.

**Effective theory:** The effective Lagrangian near the nodal-nematic transition point consists of gapless double-Weyl fermions  $\psi$ , a fluctuating  $Z_3$  order parameter  $\phi$ , and the coupling terms between them, i.e.,  $\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_\phi + \mathcal{L}_{\psi\phi}$ . Unlike Weyl points protected by translation and charge conservation symmetry, double-Weyl points need crystal symmetry protection [43, 55]. For concreteness, we consider a  $C_6$  symmetry along the  $z$ -axis to protect double-Weyl points located at  $(0, 0, \pm k_z) = \pm K$ . The  $Z_3$  nematic order breaks  $C_6$  crystal symmetry to its subgroup  $C_2$ , i.e.,  $Z_3 \cong C_6/C_2$ . Quadratic momenta doublet  $(k_x^2 - k_y^2, 2k_x k_y)$  forms a two-dimensional irreducible representation of the  $C_6$  group [56], which is denoted by  $E_2$ . Thus a simple Lagrangian for non-interacting double-Weyl fermions located at  $\pm K$  in  $k_z$ -axis reads

$$\mathcal{L}_\psi = \psi^\dagger \left( -i\omega + A[(k_x^2 - k_y^2)\sigma^x + 2k_x k_y \sigma^y] + v_{f3} k_z \sigma^z \tau^z \right) \psi, \quad (1)$$

where  $\psi = (\psi_+, \psi_-)^T$  and  $\psi_\pm$  are two-component double-Weyl fermions locating at  $\pm K$  valley, respectively.  $\sigma$  ( $\tau$ ) are Pauli matrices representing spin (valley) degrees of freedom,  $A$  is the inverse mass in  $xy$ -plane and  $v_{f3}$  is the fermion velocity along  $z$ -axis, and  $\omega$  is Matsubara frequency. The dispersion of double-Weyl fermions is anisotropic and gives rise to monopole charge  $\pm 2$  in  $k$ -space. Note that  $(\sigma^x, \sigma^y)$  is in the same  $E_2$  representation to make the Lagrangian invariant under the  $C_6$  rotation,  $\exp(i\frac{\pi}{3}\sigma^z)$ . As a simple consequence of  $C_6$  symmetry, terms, such as those constant terms proportional to  $\sigma^x, \sigma^y$ , are forbidden.

The  $Z_3$  nematic order,  $(\phi_1, \phi_2)$ , is again a doublet (two-component real boson) in  $E_2$  representation which can be described by a complex boson  $\phi \equiv \phi_1 + i\phi_2$ . A  $C_6$  rotation simply takes  $\phi$  to  $e^{i\frac{2\pi}{3}}\phi$ . Again, the coupling between double-Weyl fermions and the order parameter is dictated by symmetries, and given by

$$\mathcal{L}_{\psi\phi} = g(\phi\psi^\dagger\sigma^+\tau^z\psi + \phi^*\psi^\dagger\sigma^-\tau^z\psi), \quad (2)$$

where  $g$  is a real Yukawa coupling constant and  $\sigma^\pm = \frac{1}{2}(\sigma^x \pm i\sigma^y)$ . The effective Lagrangian for the  $Z_3$  order-parameter fields is given by

$$\mathcal{L}_\phi = |\partial_\tau \phi|^2 + v_{b\perp}^2 \sum_{i=1}^2 |\partial_i \phi|^2 + v_{b3}^2 |\partial_z \phi|^2 + r|\phi|^2 + b(\phi^3 + \phi^{*3}) + u|\phi|^4, \quad (3)$$

where  $v_{b\perp}$  and  $v_{b3}$  denote the boson velocity in the  $xy$ -plane and along the  $z$ -axis, respectively.  $r$  is a boson mass that tunes the phase transition, and  $b, u$  is strength of cubic and quartic terms, respectively. Non-vanishing

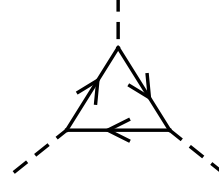


FIG. 2. Relevant Feynman diagram in generating cubic terms in Hertz-Millis-Moriya type theory. The arrowed solid lines indicate internal fermion propagator and should be integrate out. The dashed lines are external order-parameter fields.

$b$  is allowed in Eq. (3), putatively rendering a first-order transition according to the Landau criterion. (Note that a  $Z_3$  phase transition out of a topologically-ordered phase [57] can be driven by condensing fractionalized anyons whose LG theory is qualitatively different from Eq. (3).)

Owing to the non-vanishing monopole charge of a double-Weyl point, breaking rotational symmetry does not gap out the fermions. For instance, assuming  $\langle \phi \rangle = \frac{m}{gA} > 0$  in the ordered phase, the dispersion of fermions is then given by

$$E_k = \pm \sqrt{A^2[(k_x^2 - k_y^2 \pm m)^2 + 4k_x^2 k_y^2] + v_{f3}^2 k_z^2}, \quad (4)$$

from which one can deduce that the double-Weyl point at  $(0, 0, K)$  is split into two Weyl points located at  $\vec{k} = (0, \pm\sqrt{m}, K)$  and similarly for the other double-Weyl point at  $(0, 0, -K)$ .

**Poor-man methods:** In analysis of phase transitions in fermionic systems, the Hertz-Millis-Moriya (HMM) theory is often employed [19–22], where fermions presented at transitions are integrated out to obtain the effective action of order parameters. Assuming that the resulting action can be expanded in powers of order parameters with spatially local coefficients, one arrives at the Hertz model. However, such approach is shown to be incomplete due to either breakdown of Fermi-liquid theory [1, 24], or infinite number of local marginal operators generated by integrating out fermions [23, 26]. Moreover, a recent large-scale QMC study reported results not consistent with the HMM theory [27].

Remarkably, FIQCP introduced in Ref. [7] is another arena where the assumptions of the HMM theory break down, namely the effective action cannot be expanded in powers of order parameters with locally coefficients after integrating out fermions. If one insists on doing the HMM type theory, integrating out fermions cannot result in an analytic cubic term. The equivalent Feynman diagram in generating cubic terms is shown in Fig. 2 and gives rise to  $S_{\text{eff}} \ni -\Gamma_3^f \int d^4x [\phi^3 + h.c.]$ , where  $\Gamma_3^f \propto \int_k \text{Tr}[G(k)\Gamma^+ G(k)\Gamma^+ G(k)\Gamma^+]$  with a fermion propagator  $G(k)$ , vertex  $\Gamma^+ \equiv \sigma^+ \tau^z$ , and  $\int_k \equiv \int \frac{d^4k}{(2\pi)^4}$ . As a simple calculation shows that  $\Gamma_3^f = 0$ , the HMM theory *cannot* predict a FIQCP scenario.

Nonetheless, coupling to fermions can result in non-analytical terms, e.g.,  $|\phi|^3$ , of order parameter. If a non-analytical  $|\phi|^3$  term appears, it may overwhelm the cubic terms at the phase transition and drives the putative first-order transition into a continuous one [7]. To show this explicitly, we implement a poor-man method by integrating out fermions all at once and then expanding the effective free-energy as a function (possibly non-analytical) of the order-parameter. By a coordinate transformation and straightforward manipulations (see the SM for details), we find the effective free energy includes a non-analytical term:  $F_{\text{non}}[\phi] = Nb'|\phi|^3$ , where  $N$  is the flavor of four-component double-Weyl fermions and  $b'$  is a positive constant depending on the momentum cutoff. If  $N > \frac{2|b|}{b'}$ , the cubic term in the free energy has a bound  $Nb'|\phi| + b(\phi^3 + \phi^{*3}) \geq (Nb' + 2b)|\phi|^3$ , and the minimal energy is achieved from  $\phi = 0$  to nonzero continuously through phase transition. From this poor-man approach, one may expect that a continuous phase transition can occur at a putative first-order transition as long as the flavors of fermions  $N$  is sufficiently large.

**Renormalization group analysis:** We now present strong evidence of a FIQCP at the  $Z_3$  nodal-nematic transition in double-Weyl semimetals by performing standard RG analysis where fermions and bosons are treated at equal footing. The RG procedure is to integrate out fast modes to generate RG equations [1, 58]. A subtlety arises due to anisotropic dispersion of double-Weyl fermions, i.e., the scaling properties of orthogonal spatial directions are different [44, 45, 59, 60]. Here, we assume the scaling dimension for three momentum and frequency to be  $[k_z] = 1$ ,  $[k_{x,y}] = z_1$ ,  $[\omega] = z$  without loss of generality, where  $[\dots]$  denotes the scaling dimension.

The values of  $z$  and  $z_1$  are determined by RG equations of kinetic energy (see the SM for details). Letting  $\frac{v_{b3}}{v_{f3}} = 1 + \lambda$ , we assume the velocity difference is small, i.e.,  $|\lambda| \ll 1$ , the RG equations for  $\lambda$  is given by  $\frac{d\lambda}{dl} = -\Delta_\lambda \lambda$ , where  $l > 0$  is the flow parameter, and  $\Delta_\lambda$  is a positive constant independent of  $\lambda$  (see SM for details). As a result,  $\lambda = 0$  is a stable fixed point at least for small  $\lambda$ . We set  $v_{f3} = v_{b3} = v$  for simplicity. At this fixed point, the RG flow of velocity is controlled by dynamical critical exponent  $z$ , i.e.,  $\frac{d \log v}{dl} = z - 1$ . Since velocities are physical observable, they must stay finite at physical fixed point and this requires  $z = 1$  at the fixed point.

For simplicity in expressing the RG equations, we introduce four dimensionless coupling constants (not to confuse with critical exponent):

$$\beta = \frac{b^2}{\pi^2 v_{b\perp}^4 v \Lambda_\perp}, \quad \gamma = \frac{g^2}{24\pi^2 A^2 v \Lambda_\perp^2}, \quad \delta = \frac{u}{\pi^2 v_{b\perp}^2 v}, \quad (5)$$

corresponding to three running coupling constants, i.e.,  $b$ ,  $g$  and  $u$  in the interacting Lagrangian, respectively and  $\alpha = \frac{A\Lambda_\perp}{v_{b\perp}}$ . In a similar way,  $z_1$  can also be determined at the fixed point, i.e.,  $z_1 = 1 - \frac{3}{8}\beta + N(2\alpha^2 - 1)\gamma$  from the

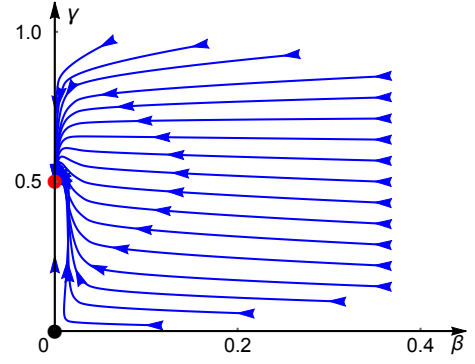


FIG. 3. The flow diagram  $\beta$ - $\gamma$  for the  $Z_3$  transition in double-Weyl semimetals for  $N = 1$  case. The arrowed curves indicate the running coupling constants as a function of energy. The red and black circles locating at  $(\frac{1}{2}, 0)$  and  $(0, 0)$  indicate a stable fixed point and Gaussian fixed point, respectively. The red one is identified as fermion induced quantum critical point. At these fixed point, one gets  $\alpha^* = \delta^* = 0$ .

RG equation for boson velocity in  $xy$ -plane,  $v_{b\perp}$  (see SM for details). Moreover, from RG of kinetic energy, the anomalous dimensions for fermions and bosons are also obtained:  $\eta_\psi = \frac{3(1-\alpha^2+\alpha^2 \log \alpha^2)}{2(1-\alpha^2)^2} \alpha^2 \gamma$  and  $\eta_\phi = N\gamma + \frac{3}{16}\beta$ .

After getting the expressions of dynamical exponents  $z$ 's and anomalous dimensions  $\eta$ 's near a physical fixed point, we are now in a position to analyze the RG equations of dimensionless coupling constants,  $(\alpha, \beta, \gamma, \delta)$ . There are two fixed points with both  $\gamma \geq 0$  and  $\delta \geq 0$ : one is the usual Gaussian fixed point and the other is a non-trivial fixed point given by  $(\alpha^*, \beta^*, \gamma^*, \delta^*) = (0, 0, \frac{1}{2N}, 0)$ , as shown in Fig. 3 where the RG flows in  $(\beta, \gamma)$ -plane are drawn (also see SM for details). As indicated in Fig. 3, the Gaussian fixed point at origin is unstable while the fixed point at  $(\beta^*, \gamma^*) = (0, \frac{1}{2N})$  is stable.

Note that  $\alpha$  approximately captures the ratio of kinetic energy between fermions and bosons in  $xy$ -plane. When the system approaches the nematic transition from disordered phase, fermion dispersion along the splitting direction becomes soft, and one can approximate the RG equations to lowest order of  $\alpha$  [31, 32]. To further justify this, the RG equations near  $\alpha = 0$  at the stable fixed point read

$$\frac{d\alpha}{dl} = -(2 + \frac{3}{2N})\alpha^3, \quad (6)$$

which shows that  $\alpha$  is irrelevant at this fixed point. Under this approximation, one gets simplified RG equations to the lowest order of  $\alpha$  near the fixed point,

$$\frac{d\beta}{dl} = (2 - 4N\gamma)\beta - \frac{3}{8}\beta^2, \quad (7)$$

$$\frac{d\gamma}{dl} = (2 - 4N\gamma)\gamma, \quad (8)$$

$$\frac{d\delta}{dl} = -(2N\gamma + \frac{5}{4}\delta)\delta. \quad (9)$$

The Gaussian fixed point is one solution of the RG equations above. However, it is unstable by perturbations along  $\beta$  and  $\gamma$  directions. There is a stable fixed point, as already indicated in the flow diagram Fig. 3, at  $(\gamma^*, \beta^*, \delta^*) = (\frac{1}{2N}, 0, 0)$ . The eigenvalues of the stability matrix are  $(0, -2, -1)$ , where the zero eigenvalue indicates a marginal direction. Indeed, one can find the deviation  $\Delta\beta$  along  $\beta$  direction is marginally irrelevant, i.e.,  $\frac{d\Delta\beta}{dt} = -\frac{3}{8}(\Delta\beta)^2$ . Thus, the nontrivial fixed point is irrelevant under perturbations along  $\gamma$  and  $\alpha$  direction and marginally irrelevant under perturbations along  $\beta$ . At this nontrivial stable fixed point, one finds that  $b^2 \propto \beta = 0$ , i.e. the cubic terms of the  $Z_3$  order-parameter is irrelevant. Consequently, this fixed point corresponds to a continuous phase transition, namely, a FIQCP! Moreover, the system has an emergent  $U(1)$  symmetry (the rotation of double-Weyl semimetal along  $z$ -axis) at the FIQCP.

We would like to emphasize that it is the presence of gapless fermions that dramatically changes the nature of the  $Z_3$  nematic phase transition. If we naively turn off fermions, i.e., set  $N = 0$ , then  $\gamma$  disappears from the RG equations of  $\beta$  and  $\gamma$ . Now the naive fixed point with  $\beta = 0$  (vanishing cubic terms) is strong relevant along  $\beta$ , which would render a first-order transition, as expected from the Landau criterion. Consequently, we expect that there should exist a critical value  $N_c$  such that a FIQCP occurs for  $N > N_c$  and a first-order transition for  $N < N_c$ . The current one-loop RG calculations shows that FIQCP occurs for any finite value of  $N$  and a more accurate value of  $N_c$  may be obtained in higher loop RG analysis in the future.

The anomalous dimensions for fermions and bosons at nontrivial fixed point are given by  $\eta_\psi = 0$  and  $\eta_\phi = \frac{1}{2}$  yielding critical exponent  $\eta = 2\eta_\phi = 1$ . Though the FIQCP is distinguished with the Gaussian fixed point, the vanishing fermion anomalous dimension implies quasiparticle picture is still valid, in contrast to the FIQCP in two-dimensional Dirac fermions [7]. Due to the validation of quasiparticle, one expects that the critical exponent  $\nu$  is given by the naive scaling argument  $\nu^{-1} = 2 + 2z_1 - 2[\phi] = 1$ , where  $z_1 = 1 - \frac{3}{8}\beta^* + N(2\alpha^{*2} - 1)\gamma^* = \frac{1}{2}$  and  $[\phi] = \frac{1}{2} + \eta_\phi = 1$  is the scaling dimension of boson field at this fixed point.

**Conclusions and discussions:** It is worth to point out again that the fluctuations of fermions at zero temperature play an essential role in a FIQCP. The three-state Potts model is a neat example featuring a  $Z_3$  transition without gapless fermion modes, where the transition is shown to be first-order in 2+1D and higher dimensions [5, 61]. However, if the transition involves large enough gapless fermions, a FIQCP may occur, realizing a type-II Landau-forbidden phase transition. The scaling dimension of the order-parameter field is often enhanced by fermions. Indeed,  $[\phi] = 1$  at the stable fixed point corresponding to the  $Z_3$  nodal-nematic transition in

double-Weyl semimetals is larger than the nominal scaling dimension of the order-parameter assigned for the first-order  $Z_3$  transitions [62]. Moreover, it is consistent with the rigorous lower bound of scaling dimension of order-parameter fields,  $[\phi] > 0.565$ , required to induce an emergent  $U(1)$  symmetry from the  $Z_3$  symmetry based on recent conformal bootstrap calculations [63]. Large anomalous dimension is also a typical feature of DQCP [8, 9, 64], where the deconfined spinons plays a similar role as gapless electrons here.

In conclusion, we present a RG study of  $Z_3$  nodal-nematic quantum phase transition in 3D topological double-Weyl semimetals, whose low-energy effective field theory contains cubic terms of order-parameters. We point out that the breakdown of Hertz-Millis-Moriya theory in describing FIQCP by a poor-man method. Unlike the Hertz-Millis-Moriya theory, our RG procedure treats fermions and bosons on equal footing the  $Z_3$  nodal-nematic transition in 3D double-Weyl semimetals and a marginal stable non-trivial fixed point is identified as a FIQCP, at which a marginal Fermi liquid theory is expected. This novel FIQCP may be observed in the future in candidate double-Weyl materials such as the one synthesized by stacking Chern insulators [65] and could shed more light to a united understanding of quantum critical phenomena.

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## SUPPLEMENTAL MATERIAL

### A. Non-analytical terms of order parameter in double-Weyl semimetals

Here we show the non-analytical terms arising by integrating out fermions explicitly. The free energy in presence of a nonzero order parameter reads

$$F[\phi] \propto -N \int d^3p \sqrt{(p_x^2 - p_y^2 + \phi)^2 + 4p_x^2 p_y^2 + p_z^2} + (\phi \rightarrow -\phi). \quad (\text{S1})$$

First we make a coordinate transformation,  $p_x = \sqrt{q \sin \theta} \cos \varphi$ ,  $p_y = \sqrt{q \sin \theta} \sin \varphi$ ,  $p_z = q \cos \theta$ , where  $q$  is a positive variable. This transformation results in a nontrivial Jacobian,  $|\frac{\partial p}{\partial q}| = \frac{q}{2}$ . After that, we get

$$F[\phi] = -\frac{1}{(2\pi)^3} \int_0^\pi d\theta \int_0^{2\pi} d\varphi \int_0^\pi dq \frac{q}{2} \sqrt{q^2 + 2q\phi \sin \theta \cos 2\varphi + \phi^2}. \quad (\text{S2})$$

The integration over  $q$  is evaluated first, this results in a complicated integral. By expanding in the order of  $\phi$ , we get the non-analytical terms

$$F[\phi] = \frac{1}{(2\pi)^3} \int d\theta d\varphi \left[ \frac{5 + 3 \cos 2\theta - 6 \cos 4\varphi \sin^2 \theta}{48} |\phi|^3 - \frac{\cos 2\varphi (3 + \cos 2\theta - 2 \cos 4\varphi \sin^2 \theta)}{16} \log(|\phi| + \cos 2\varphi \sin \theta \phi) \phi^3 \right]. \quad (\text{S3})$$

The integration can be evaluated directly,  $F[\phi] = \frac{1}{18\pi^2} |\phi|^3$ .

### B. Details for the renormalization equations at $Z_3$ nodal nematic transition of double-Weyl semimetals

The Feynman diagram Fig. S1(c) gives fermion self-energy,

$$\Sigma(p) = -\frac{1}{2} \times 2g^2 \int_k (\Gamma^+ S(k) \Gamma^- + \Gamma^- S(k) \Gamma^+) D(p-k) = \frac{g^2 l}{v_{b\perp}^2 v_{f3}} [F_\omega(-i\omega_p) + F_z v_{f3} p_z \Gamma^3], \quad (\text{S4})$$

where  $\Gamma^\pm = \sigma^\pm \tau^z$ ,  $\Gamma^3 = \sigma^z \tau^z$ ,  $k_\perp = \sqrt{k_x^2 + k_y^2}$ ,  $\int_k \equiv \int_{-\infty}^{\infty} \frac{d\omega_k}{2\pi} \int \frac{d^3 k}{(2\pi)^3}$  and  $S(k)$ ,  $D(k)$  are fermion and boson propagators, respectively. Note that in the calculation, integrations of  $\omega_k$  and  $k_z$  are not constrained, while that of momentum  $k_{x,y}$  are constrained in the momentum shell, i.e.,  $\Lambda_\perp e^{-l} < k_\perp < \Lambda_\perp$ , where  $\Lambda_\perp$  is a momentum cutoff in  $k_x k_y$ -plane and  $l > 0$  is the flow parameter. During the evaluation of Feynman diagrams, we have made a variable transformation, i.e.,  $k_{x,y} = \frac{v_{b\perp}}{A} q_{x,y}$ ,  $k_z = \frac{v_{b\perp}}{A v_{f3}} q_z$  and  $\omega_k = \frac{v_{b\perp}}{A} \omega_q$ , and it is easy to check that  $(\omega_q, \vec{q})$  are dimensionless variables.  $F_\omega$  and  $F_z$  are given by

$$F_\omega = \int_q \frac{2\omega_q^2}{l(\omega_q^2 + q_\perp^4 + q_z^2)(\omega_q^2 + q_\perp^2 + \frac{v_{b3}^2}{v_{f3}^2} q_z^2)^2} = \frac{1 - \alpha^2 + \alpha^2 \log \alpha^2}{8\pi^2(1 - \alpha^2)^2} + O(\lambda), \quad (\text{S5})$$

$$F_z = \frac{v_{b3}^2}{v_{f3}^2} \int_q \frac{2q_z^2}{l(\omega_q^2 + q_\perp^4 + q_z^2)(\omega_q^2 + q_\perp^2 + \frac{v_{b3}^2}{v_{f3}^2} q_z^2)^2} = \frac{1 - \alpha^2 + \alpha^2 \log \alpha^2}{8\pi^2(1 - \alpha^2)^2} + O(\lambda), \quad (\text{S6})$$

where  $q_\perp = \sqrt{q_x^2 + q_y^2}$  and  $\int_q = \int_{-\infty}^{\infty} \frac{d\omega_q}{2\pi} \int_{-\infty}^{\infty} \frac{dq_z}{2\pi} \int_{\alpha e^{-l}}^{\alpha} \frac{d^2 q}{(2\pi)^2}$ , and  $\alpha = \frac{A\Lambda_\perp}{v_{b\perp}}$  is the cutoff in momentum  $q_\perp$ , and  $\lambda$  is a function of  $\frac{v_{b3}}{v_{f3}}$  that will be defined below.

The boson self-energy is given by Feynman diagram Fig. S1(a) and S1(b). Evaluation of Feynman diagram Fig. S1(a) gives

$$\Pi^{(1)}(p) = \frac{g^2}{2} \int_k \text{Tr} [\Gamma^+ S(k+p) \Gamma^- S(k) + \Gamma^- S(k+p) \Gamma^+ S(k)] = \frac{4Ng^2 l}{v_{b\perp}^2 v_{f3}} [G_\omega^{(1)} \omega_p^2 + G_\perp^{(1)} v_{b\perp}^2 p_\perp^2 + G_z^{(1)} v_{b3}^2 p_z^2], \quad (\text{S7})$$

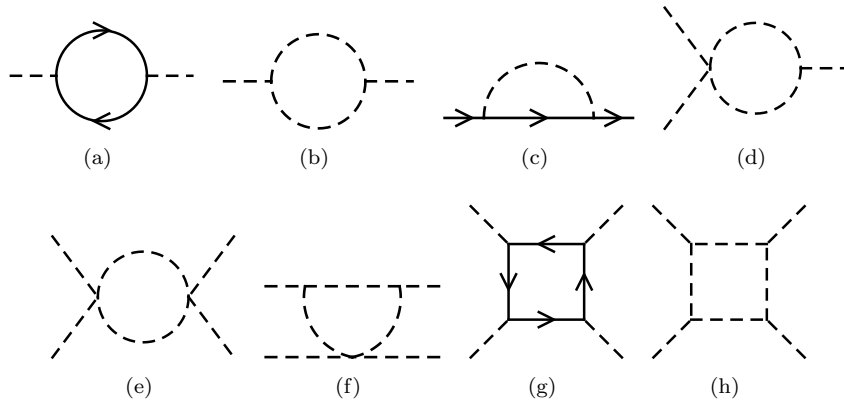


FIG. S1. One-loop Feynman diagrams. The arrowed solid line indicates fermion propagator and dashed line indicates boson propagator.

where  $\text{Tr}$  is the trace in Gamma matrices and flavor space, and  $\text{Tr}1=4N$ , where we have also promoted the flavors of four-component double-Weyl fermions to be  $N$ . Evaluation of Feynman diagram Fig. S1(b) gives

$$\Pi^{(2)}(p) = -\frac{1}{2} \times 36b^2 \int_k D(k)D(k+p) = \frac{b^2 A^2 l}{v_{b\perp}^6 v_{f3}} \left[ G_\omega^{(2)} \omega_p^2 + G_\perp^{(2)} v_{b\perp}^2 p_\perp^2 + G_z^{(2)} v_{b3}^2 p_z^2 \right]. \quad (\text{S8})$$

The the boson self-energy is  $\Pi(p) = \Pi^{(1)}(p) + \Pi^{(2)}(p)$ . Those  $G_i^{(1)}$  are given by

$$G_\omega^{(1)} = \frac{1}{4l} \int_q \left[ \frac{6\omega_q^2 + 2q_z^2}{(\omega_q^2 + q_\perp^4 + q_z^2)^3} - \frac{8\omega_q^2(q_z^2 + \omega_q^2)}{(\omega_q^2 + q_\perp^4 + q_z^2)^4} \right] = \frac{1}{48\pi^2 \alpha^2}, \quad (\text{S9})$$

$$G_\perp^{(1)} = \frac{1}{4l} \int_q \left[ \frac{8q_\perp^2(\omega_q^2 + q_z^2)}{(\omega_q^2 + q_\perp^4 + q_z^2)^3} - \frac{16q_\perp^6(q_z^2 + \omega_q^2)}{(\omega_q^2 + q_\perp^4 + q_z^2)^4} \right] = \frac{1}{24\pi^2}, \quad (\text{S10})$$

$$G_z^{(1)} = \frac{1}{4l} \int_q \left[ \frac{6q_z^2 + 2\omega_q^2}{(\omega_q^2 + q_\perp^4 + q_z^2)^3} - \frac{8q_z^2(q_z^2 + \omega_q^2)}{(\omega_q^2 + q_\perp^4 + q_z^2)^4} \right] = \frac{1}{48\pi^2 \alpha^2}, \quad (\text{S11})$$

and  $G_i^{(2)}$  are given by

$$G_\omega^{(2)} = -\frac{9}{l} \int_q \left[ \frac{8\omega_q^2}{(\omega_q^2 + q_\perp^2 + \frac{v_{b3}^2}{v_{f3}^2} q_z^2)^4} - \frac{2}{(\omega_q^2 + q_\perp^2 + \frac{v_{b3}^2}{v_{f3}^2} q_z^2)^3} \right] = \frac{3}{8\pi^2 \alpha^2} + O(\lambda), \quad (\text{S12})$$

$$G_\perp^{(2)} = -\frac{9}{l} \int_q \left[ \frac{4q_\perp^2}{(\omega_q^2 + q_\perp^2 + \frac{v_{b3}^2}{v_{f3}^2} q_z^2)^4} - \frac{2}{(\omega_q^2 + q_\perp^2 + \frac{v_{b3}^2}{v_{f3}^2} q_z^2)^3} \right] = -\frac{3}{8\pi^2 \alpha^2} + O(\lambda), \quad (\text{S13})$$

$$G_z^{(2)} = -\frac{9}{l} \int_q \left[ \frac{8\frac{v_{b3}^2}{v_{f3}^2} q_z^2}{(\omega_q^2 + q_\perp^2 + \frac{v_{b3}^2}{v_{f3}^2} q_z^2)^4} - \frac{2}{(\omega_q^2 + q_\perp^2 + \frac{v_{b3}^2}{v_{f3}^2} q_z^2)^3} \right] = \frac{3}{8\pi^2 \alpha^2} + O(\lambda). \quad (\text{S14})$$

The full set of RG equations for the various constants appearing in the kinetic energy part are given by

$$\frac{d \log A}{dl} = z - 2z_1 - \frac{g^2}{v_{b\perp}^2 v_{f3}} F_\omega, \quad (\text{S15})$$

$$\frac{d \log v_{b\perp}}{dl} = z - z_1 + \frac{\text{Tr}1 g^2}{2v_{b\perp}^2 v_{f3}} (G_\perp^{(1)} - G_\omega^{(1)}) + \frac{b^2 A^2}{2v_{b\perp}^6 v_{f3}} (G_\perp^{(2)} - G_\omega^{(2)}), \quad (\text{S16})$$

$$\frac{d \log v_{f3}}{dl} = z - 1 + \frac{g^2}{v_{b\perp}^2 v_{f3}} (F_z - F_\omega), \quad (\text{S17})$$

$$\frac{d \log v_{b3}}{dl} = z - 1 + \frac{\text{Tr}1 g^2}{2v_{b\perp}^2 v_{f3}} (G_z^{(1)} - G_\omega^{(1)}) + \frac{b^2 A^2}{2v_{b\perp}^6 v_{f3}} (G_z^{(2)} - G_\omega^{(2)}). \quad (\text{S18})$$

From above RG equations, we have

$$\frac{d \log(v_{b3}/v_{f3})}{dl} = -\frac{g^2}{v_{b\perp}^2 v_{f3}} (F_z - F_\omega) + \frac{b^2 A^2}{2v_{b\perp}^6 v_{f3}} (G_z^{(2)} - G_\omega^{(2)}). \quad (\text{S19})$$

Setting  $\frac{v_{b3}}{v_{f3}} = 1 + \lambda$ , and assuming  $\lambda \ll 1$ , a simple manipulation leads to  $\frac{d\lambda}{dl} = -\Delta_\lambda \lambda$ , where  $\Delta_\lambda \equiv \frac{g^2}{2v_{b\perp}^2 v_{f3}} H_1 + \frac{b^2 A^2}{2v_{b\perp}^6 v_{f3}} H_2$  with

$$H_1 = \int_q \frac{4q_z^2(q_\perp^2 + 3\omega_q^2 - q_z^2)}{(\omega_q^2 + q_\perp^2 + q_z^2)^3(\omega_q^2 + q_\perp^2 + q_\perp^4)} = \int \frac{dq_x dq_y}{(2\pi)^2} \int_0^\infty \frac{q_0 dq_0}{2\pi} \frac{2q_0^2(q_\perp^2 + q_0^2)}{(q_0^2 + q_\perp^2)^3(q_0^2 + q_\perp^4)}, \quad (\text{S20})$$

$$H_2 = \int_q \frac{144q_z^2(q_\perp^2 + 5\omega_q^2 - 3q_z^2)}{(\omega_q^2 + q_\perp^2 + q_\perp^4)^5} = \int \frac{dq_x dq_y}{(2\pi)^2} \int_0^\infty \frac{q_0 dq_0}{2\pi} \frac{72q_0^2(q_\perp^2 + q_0^2)}{(q_0^2 + q_\perp^2)^5}, \quad (\text{S21})$$

where we use the rotational symmetry between  $\omega_q$  and  $q_z$  in above integration to deduce that both  $H_1$  and  $H_2$  are positive. As a consequence,  $\lambda = 0$  is a stable fixed point. The RG equation for boson velocity in  $xy$ -plane  $v_{b\perp}$  reads

$$\frac{d \log v_{b\perp}}{dl} = 1 - z_1 - \frac{3}{8}\beta + N(2\alpha^2 - 1)\gamma, \quad (\text{S22})$$

where  $\alpha = \frac{A\Lambda_\perp}{v_{b\perp}}$  is also a dimensionless constant. In order to maintain the velocity, one gets  $z_1 = 1 - \frac{3}{8}\beta + N(2\alpha^2 - 1)\gamma$  at the fixed point.

Next, we calculate the remaining Feynman diagrams corresponding to coupling constant renormalizations. Feynman diagram Fig. S1(d) gives

$$\Gamma_{\phi^3} = \Gamma_{\phi^{*3}} = -\frac{3l}{4\pi^2} \frac{bu}{v_{b\perp}^2 v}. \quad (\text{S23})$$

And Feynman diagrams Fig. S1(e), S1(f), S1(g) and S1(h) give

$$\Gamma_{|\phi|^4} = -\frac{5l}{4\pi^2} \frac{u^2}{v_{b\perp}^2 v} + \frac{9l}{\pi^2} \frac{ub^2}{v_{b\perp}^4 v \Lambda_\perp^2} + \frac{Nl}{96\pi^2} \frac{g^4}{A^2 v \Lambda_\perp^2} - \frac{27l}{2\pi^2} \frac{b^4}{v_{b\perp}^6 v \Lambda_\perp^4}. \quad (\text{S24})$$

Introducing the dimensionless coupling constants, namely,  $\beta = \frac{b^2}{\pi^2 v_{b\perp}^4 v \Lambda_\perp^2}$ ,  $\gamma = \frac{g^2}{24\pi^2 A^2 v \Lambda_\perp^2}$  and  $\delta = \frac{u}{\pi^2 v_{b\perp}^2 v}$ , the RG equations read

$$\frac{d\alpha}{dl} = (-1 + \frac{3}{4}\beta + 2N\gamma)\alpha - 4N\gamma\alpha^3 + \frac{3\gamma(\alpha^2 - 1 - \alpha^2 \log \alpha^2)\alpha^3}{(\alpha^2 - 1)^2}, \quad (\text{S25})$$

$$\frac{d\beta}{dl} = (2 - 4N\gamma - \frac{3}{2}\delta)\beta - \frac{3}{8}\beta^2 - 4N\alpha^2\gamma, \quad (\text{S26})$$

$$\frac{d\gamma}{dl} = 2\gamma - 4N\gamma - \frac{9}{8}\beta\gamma + 4N\alpha^2\gamma^2, \quad (\text{S27})$$

$$\frac{d\delta}{dl} = (9\beta - 2N\gamma)\delta - \frac{5}{4}\delta^2 - \frac{27}{2}\beta^2 - 4N\alpha^2\gamma\delta + 6N\alpha^2\gamma^2. \quad (\text{S28})$$

This RG equations can be solved by a stable fixed point  $(\alpha^*, \beta^*, \gamma^*, \delta^*) = (0, \frac{1}{2N}, 0, 0)$ . By expanding the RG equations near  $\alpha = 0$ , one gets

$$\frac{d\alpha}{dl} = -(2 + \frac{3}{2N})\alpha^3, \quad (\text{S29})$$

Above RG equation shows that  $\alpha = 0$  is marginally stable at this fixed point, and as a result, we expand the RG equations in the order of  $\alpha$  as shown in the main text.

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